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CBCS SCHEME

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15MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Determine the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by applying elementary row transformations. **(05 Marks)**

b. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley Hamilton theorem. **(05 Marks)**

c. Solve by Gauss elimination method
$2x + y + 4z = 12$
$4x + 11y - z = 33$
$8x - 3y + 2z = 20$ **(06 Marks)**

2 a. Find the eigen values of $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ **(05 Marks)**

b. Solve the system of equations by Gauss elimination method.
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$ **(06 Marks)**

c. Find the rank of the matrix by reducing it to echelon form.
$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$ **(05 Marks)**

3 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ subject to $\frac{dy}{dx} = 2$, $y = 1$ at $x = 0$. **(05 Marks)**

b. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$. **(05 Marks)**

c. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \tan x$. **(06 Marks)**

4 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$. **(05 Marks)**

b. Solve $y'' + 2y' + y = 2x + x^2$ **(05 Marks)**

c. Using the method of undetermined coefficients, solve $y'' - 5y' + 6y = e^{3x} + x$ **(06 Marks)**

5 a. Find the Laplace transform of (i) $\frac{e^{-at}-e^{-bt}}{t}$ (ii) $\sin 5t \cos 2t$ **(05 Marks)**

b. Find the Laplace transform of

$$f(t)=\begin{cases} E, & 0<t<\frac{a}{2} \\ -E, & \frac{a}{2}<t<a \end{cases} \quad \text{where } f(t+a)=f(t)$$ **(06 Marks)**

c. Express $f(t)=\begin{cases} t, & 0<t<4 \\ 5, & t>4 \end{cases}$ in terms of unit step function and hence find $L[f(t)]$. **(05 Marks)**

6 a. Express $f(t)=\begin{cases} \cos t, & 0<t<\pi \\ \cos 2t, & \pi<t<2\pi \\ \cos 3t, & t>2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(06 Marks)**

b. Find the Laplace Transform of (i) $t \sin at$ (ii) $t^5 e^{4t}$ **(05 Marks)**

c. If $f(t)=t^2$, $0<t<2$ and $f(t+2)=f(t)$ for $t>2$, find $L[f(t)]$. **(05 Marks)**

7 a. Find the inverse Laplace Transform of $\frac{2s-1}{s^2+4s+29}$. **(05 Marks)**

b. Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s}{a}\right)$. **(05 Marks)**

c. Solve by using Laplace Transforms $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=e^{-t}$; $y(0)=0$, $y'(0)=0$. **(06 Marks)**

8 a. Solve the initial value problem $y''+4y'+3y=e^{-t}$ conditions with $y(0)=1$, $y'(0)=1$ using Laplace Transforms. **(06 Marks)**

b. Find the inverse Laplace Transform of $\frac{s+2}{s^2(s+3)}$ **(05 Marks)**

c. Find the inverse Laplace Transform of $\log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$ **(05 Marks)**

9 a. A box contains 3 white, 5 black and 6 red balls. If a ball is drawn at random, what is the probability that it is either red or white? **(05 Marks)**

b. The probability that a person A solves the problem is 1/3, that of B is 1/2 and that of C is 3/5. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved? **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(06 Marks)**

10 a. State and prove Baye's theorem. **(05 Marks)**

b. If A and B are events with $P(A\cup B)=\frac{3}{4}$, $P(\overline{A})=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{4}$, find P(A), P(B) and $P(A\cap \overline{B})$. **(05 Marks)**

c. Three students A, B, C, write an entrance examination. Their chances of passing are 1/2, 1/3 and 1/4 respectively. Find the probability that
(i) atleast one of them passes (ii) all of them pass (iii) atleast two of them passes. **(06 Marks)**

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